16.3 Videos Guide

16.3a

Theorem (statement and proof):

- Fundamental Theorem for Line Integrals: If C is a smooth curve and f is a differentiable function whose gradient ∇f is continuous on C, then
  $\int_C \nabla f \cdot d\mathbf{r} = f\big(\mathbf{r}(b)\big) - f\big(\mathbf{r}(a)\big)$, where $\mathbf{r}(t)$, $a \le t \le b$ describes C
  $= f(x_2, y_2, z_2) - f(x_1, y_1, z_1)$ (an analogous expression exists for the $\mathbb{R}^2$ case)

16.3b

- Description of path independence
  - $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ for any two paths C_1 and C_2 that connect the same two points

16.3c

Theorems (statement and proof):

- $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed paths $\Leftrightarrow$ $\int_C \mathbf{F} \cdot d\mathbf{r}$ is path independent
  $\Longrightarrow$ $\mathbf{F}$ is a conservative vector field (this means there exists a potential function f of $\mathbf{F}$)
  $\Longrightarrow$ $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

Exercises:

16.3d

- Find a function f such that $\mathbf{F} = \nabla f$ and (b) use part (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.
  - $\mathbf{F}(x, y) = (3 + 2xy^2)\,\mathbf{i} + 2x^2y\,\mathbf{j}$,
    C is the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $\left(4, \frac{1}{4}\right)$

16.3e

- $\mathbf{F}(x, y, z) = (y^2z + 2xz^2)\,\mathbf{i} + 2xyz\,\mathbf{j} + (xy^2 + 2x^2z)\,\mathbf{k}$,
  C: $x = \sqrt{t},\ y = t + 1,\ z = t^2,\ 0 \le t \le 1$

16.3f

- Law of Conservation of Energy (statement and proof)